Name: ______________________________

Instructor: __________________________

Math 10560, Practice Exam 1.
February 14, 2012

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- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

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1.	(a)	(b)	(c)	(d)	~~(e)~~
2.	(a)	~~(b)~~	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	~~(e)~~
4.	(a)	(b)	~~(c)~~	(d)	(e)
5.	(a)	(b)	(c)	~~(d)~~	(e)
6.	(a)	(b)	~~(c)~~	(d)	(e)
7.	(a)	(b)	~~(c)~~	(d)	(e)
8.	(a)	(b)	(c)	~~(d)~~	(e)

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Multiple Choice	________
9.	________
10.	________
11.	________
12.	________
Total	________

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Multiple Choice

1.(7 pts.) Simplify the following expression for x.

$$x = \log_3 81 + \log_3 \frac{1}{9}\ .$$

(a) $x = 9$ (b) $x = 6$ (c) $x = \ln 9 - \ln 3$

(d) $x = \ln 3$ (e) $x = 2$

$$x = \log_3 \frac{81}{9} = \log_3 9 = \log_3 (3^2) = 2$$

2.(7 pts.) The function $f(x) = x^3 + 3x + e^{2x}$ is one-to-one. Compute $(f^{-1})'(1)$.

(a) 0 (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6+e}$ (e) $\frac{1}{6+2e}$

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{5}$$

$$f^{-1}(1) = 0 \qquad f'(x) = 3x^2 + 3 + 2e^{2x}$$

$$f'(0) = 3 + 2 = 5$$

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3.(7 pts.) Differentiate the function

$$f(x) = \frac{(x^2-1)^4}{\sqrt{x^2+1}}.$$

(a) $f'(x) = \frac{x(x^2-1)^4}{\sqrt{x^2+1}}\left(\frac{8}{x^2-1} + \frac{1}{x^2+1}\right)$ $\quad f'(x)\cdot\frac{1}{f(x)} = \frac{8x}{x^2-1} - \frac{x}{x^2+1}$

(b) $f'(x) = \frac{(x^2-1)^4}{\sqrt{x^2+1}}\left(\frac{4}{x^2-1} - \frac{1}{x^2+1}\right)$

(c) $f'(x) = \frac{(x^2-1)^4}{\sqrt{x^2+1}}\left(\frac{4}{x^2-1} + \frac{1}{x^2+1}\right)$

(d) $f'(x) = \frac{(x^2-1)^4}{\sqrt{x^2+1}}\left(\frac{8}{x^2-1} - \frac{1}{x^2+1}\right)$

(e) $f'(x) = \frac{x(x^2-1)^4}{\sqrt{x^2+1}}\left(\frac{8}{x^2-1} - \frac{1}{x^2+1}\right)$

$$\ln f(x) = \ln\frac{(x^2-1)^4}{\sqrt{x^2+1}} = \ln(x^2-1)^4 - \ln(x^2+1)^{1/2}$$
$$= 4\ln(x^2-1) - \tfrac{1}{2}\ln(x^2+1)$$

4.(7 pts.) Compute the integral

$$\int_{2e}^{2e^2} \frac{1}{x(\ln\frac{x}{2})^2}\,dx.$$

(a) 2 (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) 1 (e) 0

$u = \ln\frac{x}{2} \Rightarrow du = \frac{2}{x}\cdot\frac{1}{2}dx = \frac{1}{x}dx$

$$= \int_1^2 \frac{1}{u^2}du = \frac{-1}{u}\Big|_1^2 = \frac{-1}{2} + 1 = \frac{1}{2}$$

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5.(7 pts.) Which of the following expressions gives the partial fraction decomposition of the function

$$f(x) = \frac{x^2 - 2x + 6}{x^3(x-3)(x^2+4)}?$$

~~(a) $\frac{A}{x^3} + \frac{B}{x-3} + \frac{C}{x^2+4}$~~

~~(b) $\frac{A}{x^3} + \frac{B}{x-3} + \frac{Cx+D}{x^2+4}$~~

(c) $\frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{D}{x-3} + \frac{E}{x^2+4}$

(d) $\frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{D}{x-3} + \frac{Ex+F}{x^2+4}$

~~(e) $\frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{D}{x-3} + \frac{E}{x+2} + \frac{F}{x-2}$~~

6.(7 pts.) Find $f'(x)$ if

$$f(x) = x^{\ln x}.$$

(a) $2(\ln x)x^{\ln x}$

(b) $x^{\ln x}\ln x$

(c) $2(\ln x)x^{(\ln x)-1}$

(d) $x^{\ln x}(\ln x + 1)$

(e) $x^{(\ln x)-1}\ln x$

$= e^{\ln(x^{\ln x})}$

$= e^{(\ln x)(\ln x)} = e^{(\ln x)^2}$

$f'(x) = e^{(\ln x)^2} \cdot (2\ln x) \cdot \frac{1}{x}$

$= x^{\ln x} \cdot 2\ln x \cdot x^{-1}$

$= x^{(\ln x)-1} \cdot 2\ln x$

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7.(7 pts.) Calculate the following integral.

$$\int_0^1 \frac{\arctan x}{1+x^2}\, dx\ .$$

$u = \arctan x.$

$du = \frac{1}{1+x^2} dx$

(a) $\frac{1}{2}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi^2}{32}$ (d) $\ln 2$ (e) $\frac{\pi^2}{8}$

$$\int_{\underset{\arctan(0)}{0}}^{\pi/4 = \arctan(1)} u\,du = \frac{1}{2}u^2\Big|_0^{\pi/4} = \frac{\pi^2}{32}$$

8.(7 pts.) Evaluate the integral

$$\int_0^{\pi/2} \sin^3(x)\cos^5(x) dx.$$

(a) 0 (b) $\frac{\pi}{2}$ (c) $-\frac{1}{24}$ (d) $\frac{1}{24}$ (e) $\frac{1}{4}$

$$= \int_0^{\pi/2} \sin^3(x)\cos^4(x)\cos(x)\,dx$$

$$= \int_0^{\pi/2} \sin^3(x)(1-\sin^2(x))^2\cos(x)\,dx$$

$u = \sin x$

$du = \cos x\, dx$

$$= \int_0^1 u^3(1-u^2)^2 du = \int_0^1 (u^3 - 2u^5 + u^7) du$$

$$= \frac{1}{4}u^4 - \frac{1}{3}u^6 + \frac{1}{8}u^8 \Big|_0^1 = \frac{1}{4} - \frac{1}{3} + \frac{1}{8} = \frac{6-8+3}{24}$$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

9.(11 pts.) Compute the limit

$$L = \lim_{x \to 2} \left(\frac{x}{2}\right)^{\dfrac{1}{x-2}}.$$

$$\Rightarrow \ln L = \lim_{x \to 2} \left(\frac{1}{x-2} \ln\left(\frac{x}{2}\right)\right) = \lim_{x \to 2} \frac{\ln\left(\frac{x}{2}\right)}{x-2} \begin{matrix} \to 0 \\ \to 0 \end{matrix}$$

$$\overset{H}{=} \lim_{x \to 2} \frac{\frac{1}{x}}{1} = \frac{1}{2}$$

$$\Rightarrow \boxed{L = e^{\frac{1}{2}}}$$

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10.(11 pts.) Evaluate the integral

$$\int x^2 \cos(2x)dx.$$

$$\begin{pmatrix} u=x^2 & dv=\cos 2x\,dx \\ du=2x\,dx & v=\frac{1}{2}\sin 2x \end{pmatrix} \quad = \frac{1}{2}x^2\sin 2x - \int x\sin 2x\,dx$$

$$\begin{pmatrix} u=x & dv=\sin 2x\,dx \\ du=dx & v=\frac{-1}{2}\cos 2x \end{pmatrix} \quad = \frac{1}{2}x^2\sin 2x - \left(\frac{-1}{2}x\cos 2x + \int \frac{1}{2}\cos 2x\,dx\right)$$

$$= \boxed{\frac{1}{2}x^2\sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x + C}$$

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11.(11 pts.) Evaluate:

$$\int \frac{1}{3}x^3\sqrt{9-x^2}\,dx.$$

$\frac{1}{3}x^3\sqrt{9-x^2} = \frac{1}{3}x^2\left(\sqrt{9-x^2}\right)\cdot x$

$u = 9-x^2 \Leftrightarrow x^2 = 9-u$

$du = -2x\,dx$

$= \int \frac{1}{3}(9-u)\sqrt{u}\left(\frac{-1}{2}du\right) = \frac{-1}{6}\int\left(9\sqrt{u} - u^{3/2}\right)du$

$= \frac{-1}{6}\left(6u^{3/2} - \frac{2}{5}u^{5/2}\right) + C$

$= -(9-x^2)^{3/2} + \frac{1}{15}(9-x^2)^{5/2} + C$

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12.(11 pts.) Let $C(t)$ be the concentration of a drug in the bloodstream. As the body eliminates the drug, $C(t)$ decreases at a rate that is proportional to the amount of the drug that is present at the time. Thus $C'(t) = kC(t)$, where k is a constant. The initial concentration of the drug is 4 mg/ml. After 5 hours, the concentration is 3 mg/ml.

(a) Give a formula for the concentration of the drug at time t.

$$C(t) = Me^{kt}. \quad C(0) = Me^0 = M = 4$$

$$C(5) = 4e^{5k} = 3 \Rightarrow e^{5k} = \frac{3}{4} \Rightarrow k = \frac{1}{5}\ln\frac{3}{4}$$

okay to leave here

$$\boxed{C(t) = 4e^{\left(\frac{1}{5}\ln\frac{3}{4}\right)t}} = 4e^{\frac{t}{5}\ln\frac{3}{4}} = 4e^{\ln\left(\frac{3}{4}\right)^{t/5}} = \boxed{4\cdot\left(\frac{3}{4}\right)^{t/5}}$$

(b) How much drug will there be in 10 hours?

$$C(10) = 4e^{2\ln\frac{3}{4}} = 4\cdot e^{\ln\frac{9}{16}} = 4\left(\frac{9}{16}\right) = \boxed{\frac{9}{4}}$$

$$\left(\text{or-} \quad C(10) = 4\left(\frac{3}{4}\right)^2 = 4\cdot\left(\frac{9}{16}\right) = \boxed{\frac{9}{4}}\right)$$

(c) How long will it take for the concentration to drop to 0.5 mg/ml?

$$0.5 = \frac{1}{2} = C(t) = 4e^{\left(\frac{1}{5}\ln\frac{3}{4}\right)t} \Rightarrow \frac{1}{8} = e^{\left(\frac{1}{5}\ln\frac{3}{4}\right)t}$$

$$\Rightarrow \ln\frac{1}{8} = -\ln 8 = \frac{1}{5}\ln\frac{3}{4}\,t$$

$$\Rightarrow \boxed{t = \frac{-5\ln 8}{\ln\left(\frac{3}{4}\right)}}$$

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The following is the list of useful trigonometric formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x - y) + \sin(x + y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x - y) - \cos(x + y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x - y) + \cos(x + y)\big)$$

The hyperbolic sine and cosine functions are defined to be:

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

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Multiple Choice	______
9.	______
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Math 10560, Exam 1
February 17, 2015

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2.	________
3.	________
Total	________

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Instructor: ___________________________

Multiple Choice

1.(6 pts) Which of the following expressions gives the partial fraction decomposition of the function

$$f(x) = \frac{x^2 - 2x + 6}{x^3(x-3)(x^2+4)}?$$

(a) $\dfrac{A}{x^3} + \dfrac{B}{x^2} + \dfrac{C}{x} + \dfrac{D}{x-3} + \dfrac{Ex+F}{x^2+4}$

(b) $\dfrac{A}{x^3} + \dfrac{B}{x-3} + \dfrac{C}{x^2+4}$

(c) $\dfrac{A}{x^3} + \dfrac{B}{x^2} + \dfrac{C}{x} + \dfrac{D}{x-3} + \dfrac{E}{x+2} + \dfrac{F}{x-2}$

(d) $\dfrac{A}{x^3} + \dfrac{B}{x-3} + \dfrac{Cx+D}{x^2+4}$

(e) $\dfrac{A}{x^3} + \dfrac{B}{x^2} + \dfrac{C}{x} + \dfrac{D}{x-3} + \dfrac{E}{x^2+4}$

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Instructor: ______________________

Partial Credit

You must show your work on the partial credit problems to receive credit!

2.(11 pts.) Find the integral

$$\int \frac{3x+1}{x^3+x^2}dx.$$

$$\frac{3x+1}{x^3+x^2} = \frac{3x+1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$\Rightarrow 3x+1 = Ax(x+1)+B(x+1)+Cx^2 = (A+C)x^2+(A+B)x+B$$

$$\Rightarrow \begin{cases} A + C = 0 & \to C=-2 \\ A+B = 3 & \to A=2 \\ B = 1 & \end{cases}$$

$$\int \frac{3x+1}{x^3+x^2}dx = \int \left(\frac{2}{x} + \frac{1}{x^2} - \frac{2}{x+1}\right)dx$$

$$= 2\ln|x| - \frac{1}{x} - 2\ln|x+1| + C = \boxed{\ln\left(\frac{x}{x+1}\right)^2 - \frac{1}{x} + C}$$

↑ can leave here too

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3.(11 pts.) Calculate the integral

$$\int \frac{dx}{x+\sqrt[3]{x}}.$$

$u=\sqrt[3]{x} \Leftrightarrow u^3=x$

$3u^2\,du=dx$

$$\int \frac{dx}{x+\sqrt[3]{x}} = \int \frac{3u^2}{u^3+u}\,du = \int \frac{3u}{u^2+1}\,du$$

$v=u^2+1$

$dv=2u\,du$

$$= \frac{3}{2}\int \frac{1}{v}\,dv = \frac{3}{2}\ln|v|+C$$

$$= \frac{3}{2}\ln|u^2+1|+C = \boxed{\frac{3}{2}\ln|x^{2/3}+1|+C}$$

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